

Inequalities

Linear Inequalities

Questions:

1) Solve the following inequalities:

a. $45x - 26 > 109$

b. $7x - 11 > 3x + 5$

c. $6x > 2(3x - 1)$

d. $5x + 15 < 3(x - 12) + 3x$

e. $2(x - 5) \geq \frac{1}{2}(4x + 6)$

2) Solve the following inequalities:

a. $(x - 2)^2 + 4 < (x + 2)^2 + 20$

b. $\frac{8x - 4}{2} < \frac{9(x + 1)}{3}$

c. $4(6x - 8) < 8(3x - 4)$

d. $\frac{x - 6}{3} - \frac{x - 4}{4} \geq 12 - x$

e. $\frac{7 - x}{10} - \frac{3x - 1}{5} + \frac{x + 4}{3} < 7$

3) Find the solution of the following inequalities:

a. $5x - 15 < 2(x - 10) + 3x$

b. $2(x - 5) \geq \frac{1}{2}(4x + 6)$

c. $6\left(\frac{x}{3} + 1\right) \geq 2x + 3$

d. $63\left(\frac{2x - 6}{2} + 7 - x\right) > 0$

4) Solve the following inequalities:

- | | | |
|-----------------------------|-----|----------------------------|
| a. $6x - 8 < 2x + 12$ | and | $7x - 6 > 1$ |
| b. $2(x + 4) - 3 > -7$ | and | $5x + 6 \leq 9 - (3 - x)$ |
| c. $10x < 4(x - 3)$ | and | $3x - (x - 8) > -13 - x$ |
| d. $6x - 15 - 2x < 20 - 3x$ | and | $3x + 131 \geq 3x + 131$ |
| e. $4x - 2 > 4x + 3$ | and | $5x - 3(4 + x) \leq 5 - x$ |

5) Solve the following inequalities:

- | | |
|---|---|
| a. $3 < x + 1 < 5$ | b. $6x - 38 \leq x - 3 \leq 5x + 7$ |
| c. $6(x - 1) + 3 < 7x + 14 < 4(2x - 1) + 12$ | d. $6 < \frac{2x + 10}{3} \leq \frac{7x - 20}{5}$ |
| e. $-1 \leq \frac{2x - 6}{4} < \frac{x + 2}{3}$ | |

6) Solve the following inequalities:

- | | | |
|---------------------------|----|---------------------|
| a. $3x - 6 > 18$ | or | $5x + 4 \leq 14$ |
| b. $10x + 8 \geq 6x + 20$ | or | $4x + 8 < x - 10$ |
| c. $5x + 9 > 30 - 2x$ | or | $6 - x < x + 4$ |
| d. $4x + 8 > x - 10$ | or | $x - 10 < 9x + 14$ |
| e. $4x - 6 > 14 - x$ | or | $5x - 11 < 19 + 2x$ |

Final Answers:

- | | | | | |
|-----------------------------|---------------------------|------------------|----------------|----------------|
| 1) a. $x > 3$ | b. $x > 4$ | c. All x | d. $x > 51$ | e. No solution |
| 2) a. $x > -2$ | b. $x < 5$ | c. No solution | d. $x \geq 12$ | e. $x > -13$ |
| 3) a. No solution | b. No solution | c. All x | d. All x | |
| 4) a. $1 < x < 5$ | b. $-6 < x \leq 0$ | c. $-7 < x < -2$ | | |
| d. All x | e. No such x | | | |
| 5) a. $2 < x < 4$ | b. $-2.5 \leq x \leq 7$ | c. $x > 6$ | d. $x \geq 10$ | |
| e. $1 \leq x \leq 13$ | | | | |
| 6) a. $x > 8$ or $x \leq 2$ | b. $x \geq 6$ or $x < -6$ | c. $x > -5$ | d. $x > -6$ | |
| e. All x . | | | | |

Polynomial Inequalities

Questions:

1) Solve the following inequalities:

a. $x^2 < 144$

b. $-3x^2 + 12x > 0$

c. $x^2 - 10x + 25 > 0$

d. $x^2 - 12x > -32$

2) Solve the following inequalities:

a. $2x^2 + 2x + 24 \geq 0$

b. $x^2 + x + 24 \geq 0$

c. $-x^2 + 13x + 30 < 0$

d. $-2x^2 + 7x + 4 < 0$

3) Solve the following inequalities:

a. $(x+2)(x+5) < 0$

b. $(x+2)(x+4) < 35$

c. $(x-3)(x-7) \geq 8x - 56$

d. $(x-3)^2 > (x-1)(x+6) - x^2 - 3x$

4) Solve the following sets of inequalities:

a. $x^2 - 7x + 6 \leq 0$

and

$3x - 1 < 11$

b. $x^2 - 8x + 15 > 0$

and

$x^2 - 36 < 0$

c. $x^2 - 19x + 90 \geq 0$

or

$x^2 - x - 2 \leq 0$

d. $x^2 - 3x - 4 < 0$

or

$x^2 - 1 > 0$

5) Solve the following inequalities:

a. $(x-1)(x-2)(x-3) > 0$

b. $x(x^2 + x + 1) > 0$

c. $(-2x^2 - 3x + 2)(x+1) \leq 0$

d. $(x^2 + 6)(x+3) > 0$

e. $(x^2 + 8x + 20)(3x - 5) \leq 0$

6) Solve the following inequalities:

a. $x^3 - 25x \geq 0$

b. $x^4 > x^2$

c. $x^3 - 6x^2 + 9x \leq 0$

d. $4x^3 - 4x^2 + x \leq 0$

e. $3x^4 + 4x^3 > -x^2$

7) Solve the following sets of inequalities:

a. $3x+1 > 0$ and $(x-2)(2x-1)(3x-2) < 0$

b. $x^2 < 2x$ or $(x-3)(x-6)(x-9) > 0$

Final Answers:

1) a. $-12 < x < 12$ b. $0 < x < 4$ c. $x < 5$ or $x > 5$ d. $x < 4$ or $x > 8$

2) a. All x b. All x c. $x < -2$ or $x > 15$ d. $x < -\frac{1}{2}$ or $x > 4$

3) a. $-5 < x < -2$ b. $-9 < x < 3$ c. $x \leq 7$ or $x \geq 11$ d. $x < 3$ or $x > 5$

4) a. $1 \leq x < 4$ b. $-6 < x < 3$ or $5 < x < 6$ c. $x \leq 9$ or $x \geq 10$
d. $x < -1$ or $x > -1$

5) a. $1 < x < 2$ or $x > 3$ b. $x > 0$ c. $-2 \leq x \leq -1$ or $x \geq \frac{1}{2}$

d. $x > -3$ e. $x \leq \frac{1}{2}$

6) a. $-5 \leq x \leq 0$ or $x \leq 5$ b. $x < -1$ or $x > 1$ c. $x \leq 0$ or $x = 3$

d. $x \leq 0$ or $x = \frac{1}{2}$ e. $x < -1$ or $-\frac{1}{3} < x < 0$ or $x > 0$

7) a. $-\frac{1}{3} < x < \frac{1}{2}$ or $\frac{2}{3} < x < 2$ b. $0 < x < 2$ or $3 < x < 6$ or $x > 9$

Rational Inequalities

Questions:

1) Solve the following inequalities:

a. $\frac{2x-1}{x-5} \leq 0$

b. $\frac{1}{x^2-16} > 0$

c. $\frac{1}{-3(x-1)} < 0$

d. $\frac{x-1}{x^2-9} > 0$

e. $\frac{5-2x}{(x-8)^2} \leq 0$

2) Solve the following inequalities:

a. $\frac{(x-4)(x+2)}{x-1} < 0$

b. $\frac{1}{x^2-8x+12} \geq 0$

c. $\frac{x-3}{2x^2-10x+12} > 0$

d. $\frac{(x-5)(3x+1)}{(2-x)(x+7)} < 0$

e. $\frac{(x-6)^2(x+1)}{x-2} > 0$

3) Solve the following inequalities:

a. $\frac{x-1}{3x+2} \geq -3$

b. $\frac{x-3}{x-2} < 2$

c. $\frac{x-3}{x^2-3x+2} < 3$

d. $\frac{2x+7}{x^2-6x+8} \geq 1$

e. $\frac{7x^2+1x}{3x-4} \geq 2x-10$

4) Solve the following inequalities:

a. $\frac{25}{x^2} - \frac{x+7}{x} \geq \frac{3}{x} - 2$

b. $\frac{x-10}{x} + \frac{2}{3} < \frac{x-6}{x-2}$

5) Solve the following inequalities:

a. $\frac{2x+7}{5-x} < 7 - \frac{6x}{x+3}$

b. $\frac{3}{3x-2} - \frac{1}{4x+1} \geq \frac{2x}{9x-6}$

6) Solve the following inequalities:

a. $\frac{x+3}{x} + \frac{x-4}{5} \leq \frac{x^2-4x+4}{5x-20}$

b. $\frac{x}{x^2-4} + \frac{1}{x+2} < \frac{1}{x-2}$

7) Solve the following inequalities:

a. $\frac{4}{2x+4} + \frac{3}{3x-6} < \frac{4}{x^2-4}$

b. $\frac{x}{x+5} - \frac{1}{x^2+x-20} > \frac{1}{x-4}$

8) Solve the following inequalities:

a. $\frac{27}{x^2-6x+9} + \frac{10x}{3x-9} < 3$

b. $\frac{x-7}{x-1} - \frac{3x}{4x+28} \geq \frac{3}{x^2+6x-7}$

Final Answers:

1) a. $\frac{1}{2} \leq x < 5$

b. $x < -4$ or $x > 4$ c. $x > 1$

d. $-3 < x < 1$ or $x > 3$

e. $2\frac{1}{2} \leq x < 8$ or $x > 8$

2) a. $x < -2$ or $1 < x < 4$

b. $x < 2$ or $x > 6$ c. $x > 2$ and $x \neq 3$

alternative: $2 < x < 3$ or $x > 3$

d. $x < -7$ or $-\frac{1}{3} < x < 2$ or $x > 5$

e. $x < -1$ or $(x > 2$ and $x \neq 6)$

3) a. $x < -\frac{2}{3}$ or $x \geq -\frac{1}{2}$

b. $x < 1$ or $x > 2$ c. $x < 1$ or $x > 2$

d. $1 \leq x < 2$ or $4 < x \leq 7$

e. $-40 \leq x \leq 1$ or $x \geq \frac{4}{3}$

4) a. $x \neq 0$

b. $0 < x < 2$ or $5 < x < 6$

5) a. $x < -12$ or $-3 < x < 2\frac{1}{3}$ or $x > 5$

b. $-\frac{1}{2} \leq x < -\frac{1}{4}$ or $\frac{2}{3} < x \leq 2\frac{1}{2}$

6) a. $-12 \leq x < 0$ or $4 < x \leq 5$

b. $x < -2$ or $2 < x < 4$

7) a. $x < -2$

b. $x < -5$ or $-1 < x < 4$ or $x > 6$

8) a. $x < -24$ or $0 < x < 3$ or $x > 3$

b. $x \leq -16$ or $-7 < x < 1$ or $x > 13$

Absolute Value Inequalities

Questions:

1) Solve the following sets of inequalities:

a. $|x-4| < 5$

b. $|x+3| < 7$

c. $|3x-6| < 18$

d. $|5x+16| < 41$

2) Solve the following sets of inequalities:

a. $|14-x| < 7$

b. $|12-x| < x$

c. $|x^2-x| < 6$

d. $|x^2+2x| < 8$

3) Solve the following sets of inequalities:

a. $|2x+1| < 7$ and $|x+1| > 3$

b. $|x+2| < x$ and $|x+3| > 1$

c. $|x^2+1| > 2$ and $|x^2-2x| < 3$

Final Answers:

- 1) a. $-1 < x < 9$ b. $-10 < x < 4$ c. $-4 < x < 8$ d. $-11\frac{2}{5} < x < 5$
- 2) a. $7 < x < 21$ b. $x > 6$ c. $-2 < x < 3$ d. $-4 < x < 2$
- 3) a. $2 < x < 3$ b. No such x c. $1 < x < 3$