

# Word Problems

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- 1) For a given curve, the slope of the tangent at each point  $(x, y)$  on the curve is equal to  $-\frac{x}{y}$ .  
Find the equation of the curve.
- 2) Given a curve, in the first quadrant, which goes through the point  $(1, 3)$ , and that the slope of its tangent at the point  $(x, y)$  equals  $-\left(1 + \frac{y}{x}\right)$ . Find the equation of the curve.
- 3) Find the equation of the curve, whose normal at each point passes through the origin.
- 4) Find the equation of the curve, the slope of whose tangent at each point is equal to half the slope of the segment from the origin to the point.
- 5) Find the equation of the curve which passes through the point  $(1, 2)$  and, for each point  $(x, y)$  on it, the slope of the normal is  $\frac{2xy}{y^2 - x^2}$ .
- 6) Given a curve in the first quadrant, passing through the point  $(2, 4)$ . Also given that for each point  $A(x, y)$  on it, the difference between the slope of the tangent to the curve at  $A$  and between the slope of the line connecting  $A$  with the origin, is equal to the  $y$ -coordinate of  $A$ . Find the equation of the curve.
- 7) Find the equation of the curve that passes through the origin and which is perpendicular to each line connecting a point on the curve to the point  $(3, 4)$ .
- 8) The area  $S$  is bounded the curve  $y = y(x)$ , the  $x$ -axis and the lines  $x = a$ ,  $x = x$  (variable); see diagram below. It is known that the area  $S$  is proportional to the arc length between the points  $(a, y(a))$  and  $(x, y(x))$ . Find the equation of the curve.
- 9) Find the family of curves orthogonal to the family  $\{x + 2y = c\}$ .

- 10) Find the family of curves orthogonal to the family  $\{xy = c\}$ .
- 11) Answer the following questions:
- Find the family of curves orthogonal to the family  $\{x^2 + 2y^2 = c\}$ .
  - Find the curve orthogonal to the curve  $x^2 + 2y^2 = 9$  at the point  $(1, 2)$  on it.
- 12) Find the family of curves orthogonal to the family  $\{x^2 + y^2 = cx\}$ .
- 13) Find the family of curves which form a  $45^\circ$  angle with the family  $\{x^2 + y^2 = c\}$ .
- 14) At each point on a curve the segment of the normal between the point and the x-axis is bisected by the y-axis. Find the equation of the curve.
- 15) Find the equation of the curve passing through the point  $(0, 1)$  such that the triangle bounded by the y-axis, the tangent to the curve at any point  $M(x, y)$  on it, and the segment OM from the origin O to M, is an isosceles triangle whose base is the segment MN, where N is the intersection of the tangent with the y-axis. Illustrate the problem with a sketch in the first quadrant.
- 16) The area S is bounded the curve  $y = y(x)$  the  $x$ -axis and the lines  $x = 1$ ,  $x = x$  (variable); See diagram below. It is known that  $y(1) = 2$ . Does such a curve exist such that the area of S equals  $2y(x)$ ?
- 17) The area S is bounded the curve  $y = y(x)$ , the x-axis and the lines  $x = 1$ ,  $x = x$  (variable); See diagram below. It is known that  $y(1) = 2$ . Does such a curve exist such that the area of S equals  $y(x) - 2$ ?
- 18) The area S is bounded the curve  $y = y(x)$ , the x-axis and the lines  $x = 1$ ,  $x = x$  (variable); See diagram below. It is known that  $y(1) = 2$ . Does such a curve exist such that the area of S equals  $y(x) - 2$ ?
- 19) Given a curve passing through the point  $B(0, 1)$ . At each point A on the curve, the slope is equal to the area of the trapezoid ABCD as shown in the picture. What is the equation of the curve?

- 20)** If a quantity  $y(t)$  grows [decays] exponentially; i.e. at each instant the rate of growth [decay] is proportional to its value. Suppose that at the start time  $t = 0$  the quantity is  $y_0$  and that the constant of proportionality is  $k$ . Find a formula for the quantity at any time  $t$ .
- 21)** The population of the earth is increasing at a rate of 2% per year. It was found to be 4 billion in 1980.
- What will the population of the earth be in 2010?
  - What was the population of the earth in 1974?
  - When will a population of 50 billion be reached?  
(Assume that the population is growing exponentially; i.e. at each instant the rate of growth is proportional to its value).
- 22)** The population in a certain city grows exponentially. In a certain year there were 400 thousand residents and 4 years later there were 440 thousand.
- Find the annual growth rate (as a %).
  - After how many years (from that certain year) were there 550 thousand residents?
- 23)** A man deposited money in the bank at an interest rate of 4% compounded annually. After 5 years he had accumulated \$5000.
- How much did he initially deposit?
  - After how many years will he have accumulated \$7000?
- 24)** The number of wild animals at a nature reserve grows exponentially. There were 1000 animals at the initial count. At a second count, 20 months later, there were 1400 wild animals. How many months after the initial count will the reserve have 2000 animals?
- 25)** The radioactive isotope carbon-14 has a half-life of 5750 years. At any given moment its rate of decay is proportion to the amount present.
- How many grams of this isotope will survive after 1000 years, if there were 100 grams initially?
  - After how many years will there remain just 10 grams of the initial 100 grams?

## Ordinary Differential Equations

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- 26)** In a certain pool there are 240 tons of fish, and the quantity of fish in it increases by 4% each week. In a second pool there are 200 tons of fish, and the quantity of fish in it increases by 10% each week.
- After how many weeks will both pools have the same quantity of fish?
  - After how many weeks will the second pool have twice the quantity of fish as the first pool?
- 27)** At time  $t = 0$  a tank contains 4 kg of salt dissolved in 200 liters of water. Salt water, at a concentration of 0.2 kg per liter of water, is flowing into the tank at a rate of 25 liters per minute and, simultaneously, the mixed solution is draining out of the tank at the same rate.
- Compute the amount of salt in the tank after 8 minutes.
  - After how long will the amount of salt in the tank be twice the initial amount?
- 28)** A rowboat is initially towed at a rate of 12 km/h. At time  $t = 0$  the cable is released and a man in the boat starts rowing in the direction of the motion, and applies a force of 20 newton to the boat. The mass of the boat & rower is 500 kg and the resistance (newton) is  $2v$ , where  $v$  is the velocity of the boat in meters/sec.
- 29)** Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and temperature of its surroundings. A substance with a temperature of  $150^{\circ}\text{C}$  is in a container which has the surrounding temperature of the air, a constant  $30^{\circ}\text{C}$ . The substance cools in accordance with Newton's Law of Cooling and, after half an hour, its temperature drops to  $70^{\circ}\text{C}$ .
- What is its temperature after an hour?
  - After how long will its temperature be  $40^{\circ}\text{C}$ ?
- 30)** A spring of negligible weight is suspended vertically. A mass  $m$  is connected to its free end. If the mass is moving at a velocity  $v_0$  m/sec when the spring is not extended, find the velocity  $v$  (in m/sec) as a function of the spring's extension (in m).

### Final Answers:

1)  $y^2 + x^2 = k$

3)  $x^2 + y^2 = k$

5)  $x^3 - 3y^2x = 11$

7)  $y = 4 \pm \sqrt{25 - (x-3)^2}$

9)  $y = 2x + k$

11) a.  $y = ax^2$                       b.  $y = 2x^2$

12)  $y = m(x-c)^2, y > 0$

14)  $2x^2 + y^2 = k$

16)  $4e^{-0.5}e^{0.5x} \Big|_1^x \neq 4e^{-0.5}e^{0.5x}$

18)  $2e^{x-1} - 2 = 2e^{x-1} - 2$

20)  $y(t) = y_0e^{kt}$

21) a. 7.28                      b. 4.51                      c. 126, year 2016

22) a.  $400e^{0.02t}$                       b. 15.92<sub>years</sub>

23) a.  $y(t) = 4093.65 \cdot e^{0.04t}$                       b. 13.41<sub>years</sub>

24) 40.77<sub>months</sub>

25) a. 88.69<sub>gr</sub>                      b. 19188<sub>years</sub>

26) a. 3.04<sub>weeks</sub>                      b. 14.6<sub>weeks</sub>

27) a. 26.75<sub>kg</sub>                      b. 0.942<sub>min</sub>

28) 10<sub>m/sec</sub>

29) a.  $43\frac{1}{3}$                       b. 1.13<sub>hours</sub>

30)  $v = \pm \sqrt{2gx - \frac{kx^2}{m} + v_0^2}$

2)  $2yx + x^2 = 7$

4)  $y^2 = ax$

6)  $y = 2e^{-2}xe^x = 2xe^{x-2}$

8)  $y = k \cosh\left(\pm \frac{1}{k}x + C\right)$

10)  $y^2 - x^2 = k$

13)  $\ln|x| + \frac{1}{2} \ln\left(\left(\frac{y}{x}\right)^2 + 1\right) = -\arctan\left(\frac{y}{x}\right) + c$

15)  $2 = y + \sqrt{y^2 + x^2}$

17)  $4e^{-0.5}e^{0.5x} \Big|_1^x \neq 4e^{-0.5}e^{0.5x}$

19)  $y = 2e^{x^2/4} - 1$