

Polynomial Functions

Polynomial Division

Questions:

1) Use long division to perform the following division problems:

a. Divide $4x^3 + 5x^2 - x - 11$ by $4x^2 + x - 6$

b. Divide $3x^4 + 10x^3 + 17x^2 + 10x + 24$ by $3x^2 + 14x + 8$

2) Use long division to evaluate the following problems:

a.
$$\frac{x^5 - 8x^4 + 15x^3 + 20x^2 - 76x + 48}{x^2 - x - 6} = ?$$

b.
$$\frac{x^5 - 8x^4 + 15x^3 + 20x^2 - 76x + 48}{x^3 - 2x^2 - 5x + 6} = ?$$

3) Use long division to solve the following division problems:

a. Divide $3x^3 + 3x^2 + 3x + 1$ by $x + 6$

b. Divide $x^3 + 12x^2 + 10x + 8$ by $4x^2 + 3x + 1$

4) Use long division to solve the following division problems:

a. Divide $x^4 + 4x^3 + 3x^2 - 4x - 3$ by $x^2 + 10x - 9$

b. Divide $3x^4 - 12x^3 + 8x^2 - 10x + 2$ by $x^3 + x^2 + 7x + 1$

5) Use long division to solve the following division problems:

- a. Divide $x^3 + x^2 - 10x + 8$ by $x^2 + 3x - 4$
 b. Divide $2x^4 - x^3 - 14x^2 + 19x - 6$ by $2x^2 - 3x + 1$

6) Use long division to evaluate the following:

- a.
$$\frac{2x^6 - 13x^5 - 31x^3 + 31x^4 + 8x - 4 + 7x^2}{(2x+1)(x-2)}$$

 b.
$$\frac{2x^7 + 25x^4 + 120x^3 + 270x + 270x^2 + 81}{(x-1)^2}$$

7) Use long division to evaluate the following:

- a.
$$\frac{x^4 - ax^3 + x - a}{x - a} = ?$$

 b.
$$\frac{x^3 + (1-b)x^2 - (2+b)x + 2b}{x - 2 + x^2} = ?$$

Final Answers:

- | | |
|---|---|
| 1) a. $x + 1 + \frac{4x - 5}{4x^2 + x - 6}$ | b. $x^2 - \frac{4}{3}x + 9\frac{2}{9} - \frac{108\frac{4}{9}x + 49\frac{7}{9}}{3x^2 + 14x + 8}$ |
| 2) a. $x^3 - 7x^2 + 14x - 8$ | b. $x^2 - 6x + 8$ |
| 3) a. $3x^2 - 15x + 93 - \frac{557}{x + 6}$ | b. $4x^2 + 3x + 1 + \frac{21x + 83}{16(4x^2 + 3x + 1)}$ |
| 4) a. $x^2 - 6x + 72 + \frac{-778x + 645}{x^2 + 10x - 9}$ | b. $3x - 15 + \frac{2x^2 + 92x + 17}{x^3 + x^2 - 7x + 1}$ |
| 5) a. $x - 2$ | b. $x^2 + x - 6$ |
| 6) a. $x^4 - 5x^3 + 9x^2 - 7x + 2$ | b. $2x^5 + 4x^4 + 6x^3 + 33x^2 + 80x - 43$ |
| 7) a. $x^3 + 1$ | b. $x - b$ |

Polynomial Roots

Questions:

1) Solve the equations:

a. $x^3 - 2x^2 + x - 2 = 0$

b. $x^3 + 2x^2 - 5x - 6 = 0$

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b. $x^4 - x^3 - 7x^2 + x + 6 = 0$

c. $x^4 - x^3 - 11x^2 + 9x + 18 = 0$

d. $x^3 - 7x^2 + 14x - 8 = 0$

e. $x^3 + x^2 - 17x + 15 = 0$

2) Solve the equations:

a. $2x^3 + 3x^2 - 8x + 3 = 0$

b. $2x^3 + x^2 - 2x - 1 = 0$

c. $4x^3 + 5x^2 - 7x - 2 = 0$

3) Solve the equations (knowledge about the derivative is needed):

a. $x^3 + x^2 - 5x + 3 = 0$

b. $x^3 - 3x - 2 = 0$

c. $x^5 - 3x^4 - 5x^3 + 27x^2 - 32x + 12 = 0$

d. $x^3 - 3x^2 + 3x - 1 = 0$

4) Answer the following questions:

a. Solve the equation $x^3 - x^2 - 2x + 2 = 0$.

b. Find all the zeroes of the polynomial $p(x) = x^3 - x^2 - 2x + 2$.

5) Answer the following questions:

a. Solve the equation $x^3 - 4x^2 + 5x - 2 = 0$.

b. Find all the zeroes of the polynomial $f(x) = x^3 - 4x^2 + 5x - 2$.

6) Solve the equations:

a. $x^3 - x^2 - x - 2 = 0$

b. $x^4 - x^3 - 5x^2 + 3x + 6 = 0$

7) For each polynomial, list all its zeros and give their multiplicities:

c. $p(x) = x^4 + 3x^3 + 3x^2 - x - 6$

d. $q(x) = x^5 - 5x^4 + 9x^3 - 9x^2 + 8x - 4$

Final Answers:

- 1) a. $x=2$ $z=2, \pm i$ b. $x=-1, 2, -3$ c. $x=1, -2, 3$ d. $x=\pm 1, -2, 3$
 e. $x=-1, 2, \pm 3$ f. $x=1, 2, 4$ g. $x=1, 3, -5$
- 2) a. $x=1, 3, -\frac{1}{2}$ b. $x=-1, -\frac{1}{2}$ c. $x=-\frac{1}{4}, -2$
- 3) a. $x=1, -3$ b. $x=-1, 2$ c. $x=1, 2, -3$ d. $x=1$
- 4) a. $x=1, \pm\sqrt{2}$ b. $x=1, \pm\sqrt{2}$
- 5) a. $x=1, 2$ b. $x=1, 2$
- 6) a. $x=2$, $z=-\frac{1}{2}+\frac{\sqrt{3}}{2}i, -\frac{1}{2}-\frac{\sqrt{3}}{2}i$ b. $x=-1, 2, \pm\sqrt{3}$
- 7) a. $x=1, -2$, $z=-1\pm\sqrt{2}i$ b. $x=1, 2$, $z=\pm i$

Partial Fractions

Questions:

Determine the partial fractions decomposition of the following expressions:

1) $\frac{1}{x^2-4}$

2) $\frac{5-x}{x^2+5x}$

3) $\frac{x}{x^2+5x+6}$

4) $\frac{8x-1}{2x^2-3x-2}$

5) $\frac{x+4}{(x-1)^2}$

6) $\frac{6-x}{x^2+8x+16}$

7) $\frac{x^2+x-1}{x^3-x}$

8) $\frac{10x}{x^4-13x^2+36}$

9) $\frac{8x}{(x-2)^2(x+2)}$

10) $\frac{5-x}{x^3+x^2}$

11) $\frac{9x+36}{x^3+6x^2+9x}$

12) $\frac{1}{(x^2-2x+1)(x^2-4x+4)}$

13) $\frac{x+4}{(x-1)^3}$

14) $\frac{6x^2-4x+1}{(x-1)^3}$

15) $\frac{2x^2+2x+1}{(x^2+1)(x+2)}$

16) $\frac{2x^2+x-1}{(x^2+1)(x-3)}$

17) $\frac{3}{(x^2+1)(x^2+4)}$

18) $\frac{1}{x(x^2+1)^2}$

Final Answers:

1) $\frac{1/4}{x-2} - \frac{1/4}{x+2}$

2) $\frac{1}{x} - \frac{2}{x+5}$

3) $\frac{3}{x+3} - \frac{2}{x+2}$

4) $\frac{2}{2x+1} + \frac{3}{x-2}$

5) $\frac{1}{x-1} + \frac{5}{(x-1)^2}$

6) $-\frac{1}{x+4} + \frac{10}{(x+4)^2}$

7) $\frac{1}{x} + \frac{1/2}{x-1} - \frac{1/2}{x+1}$

8) $\frac{1}{x+3} + \frac{1}{x-3} - \frac{1}{x+2} - \frac{1}{x-2}$

9) $\frac{1}{x-2} + \frac{4}{(x-2)^2} - \frac{1}{x+2}$

10) $-\frac{6}{x} + \frac{5}{x^2} + \frac{6}{x+1}$

11) $\frac{4}{x} - \frac{4}{x+3} - \frac{3}{(x+3)^2}$

12) $\frac{2}{x-1} + \frac{1}{(x-1)^2} - \frac{2}{x-2} + \frac{1}{(x-2)^2}$

13) $\frac{1}{(x-1)^2} + \frac{5}{(x-1)^3}$

14) $\frac{6}{x-1} + \frac{8}{(x-1)^2} + \frac{3}{(x-1)^3}$

15) $\frac{x}{x^2+1} + \frac{1}{x+2}$

16) $\frac{1}{x^2+1} + \frac{2}{x-3}$

17) $\frac{1}{x^2+1} - \frac{1}{x^2+4}$

18) $\frac{1}{x} - \frac{x}{x^2+1} - \frac{x}{(x^2+1)^2}$